Introduction

- Consensus formation models are useful for obtaining harmonious consensus in a group when all group members pursue their individual preferences.
- In the process of consensus formation, the agent accepts macro-information and adapts its preference to the overall group preference.
- A technique that uses computer-based, group decision-making support systems (GDSS) is an important approach to solving this problem.
- Analytic Hierarchy Process (AHP) and Interpretive Structural Modeling (ISM) were particularly used for solving the problem.
Purpose

- We develop Flexible ISM (FISM). FISM is a method for developing structural models of complex systems.
- To perform FISM logically and effectively, we propose a partially filled reachability matrix and implication rules.
- In this paper, we propose consensus formation models based on FISM with a consensus matrix.
- We propose two agent's strategies for consensus formation with an associated matrix, and analyze some quantitative measures of the difference between the individual preferences and the consensus.
Contents

- Group Decision Making Support System
  - Consensus Formation Model
  - Flexible Structural Modeling: FISM

- Consensus Formation Process based on FISM
  - Consensus Formation Process
  - Consensus Strategy
  - Evaluation

- Simulation

- Result and Conclusion
Basic models

An individual $k$ has a matrix $M^{(k)} (k = 1, 2, \ldots, K)$ of the set of alternatives as his own mental model.

Model

- $W = \{O_1, O_2, \ldots, O_N\}$
- $M = \{M^{(k)} | k = 1, 2, \ldots, K\}$
- $M^{(k)} = [m_{ij}^k] (i \in W, j \in W)$

\[
M^{(k)} = \begin{cases} 
1 & \text{if } O_i \succeq^{(k)} O_j \\
0 & \text{otherwise}
\end{cases}
\]

$W$ is a set of alternatives.

$M$ is a relational matrix.

$M$ is antisymmetric, reflexivity and transitivity. $M$ is a reachability matrix.
Consensus formation may be defined in the form of the following problems. “For a certain group producing possible arrival matrixes $M^{(1)}$, $M^{(2)}$, $\cdots$, $M^{(n)}$. The process is aimed at constructing one reachability matrix $C$ under the consensus of a group”.

Consensus Matrix $C = M^{(1)} = M^{(2)} = \cdots = M^{(n)}$
A consensus formation process based on FISM

1. Construct a matrix $M^{(k)}$ of preference relations of an individual in a group.
2. Construct the consensus matrix $C$.
3. Repeat the following steps until $C$ becomes a known matrix.
   - Select an unknown element ($c_{ij} = x$)
   - Input a value of 0 or 1 for $c_{ij}$
   - Update the matrix $C$ using implication rules and the value of $c_{ij}$
4. As a result, the matrix $C$ becomes a consensus reachability matrix.
A consensus formation process (1)

1. A matrix $M^{(k)}$ of the preference relations of an individual in the group is constructed.

$$M = \left\{ M^{(1)}, M^{(2)}, \ldots, M^{(K)} \right\}$$

2. The consensus matrix $C$ can invent a group personal difference.

$$C = [c_{ij}]$$

$$c_{ij} = \begin{cases} m_{ij}, & \text{if } m_{ij}^{(1)} = m_{ij}^{(2)} = \cdots = m_{ij}^{(K)} \\ x & \text{otherwise} \end{cases}$$
A consensus formation process (2)

3. Repeat the following steps until C becomes a known matrix.
   - Select an unknown element \( (c_{ij} = x) \)
   - Input a value of 0 or 1 for \( c_{ij} \)
   - Update the matrix C using implication rules and the value of \( c_{ij} \)

4. As a result, the matrix C becomes a consensus reachability matrix.
Evaluation of a consensus

- group consensus degree: AFI
  - AFI are defined as demonstrating how well each person agrees with the group.
    
    \[
    \begin{align*}
    \text{DI}^{(k)} &= \begin{bmatrix} d_{ij}^k \end{bmatrix} \\
    d_{ij}^k &= m_{ij}^k - c_{ij} \\
    \text{FI}^{(k)} &= 1 - \frac{1}{n(n-1)} \sum_{i=1}^{N} \sum_{j=1}^{N} |d_{ij}^k| \\
    \text{AFI}(M, C) &= \frac{1}{K} \sum_{k=1}^{K} \text{FI}^{(k)}
    \end{align*}
    \]

- consensus arrival rate
  - The consensus arrival rate is defined as the update count divided by \(N(N - 1)\), which is the number of elements in the matrix excluding diagonal elements.
Consensus strategy simulation

- In the consensus formation, there is a problem with how individuals come to agreement on topics on which opinion the value of elements group discussion during consensus formation.

- The following strategies must be considered for effectively simulating consensus formation.
  - A strategy for selecting unknown elements \((i, j)\) of matrix C
  - A strategy for inputting a value for \(c_{ij}\)

- The effectiveness of consensus formation is examined using two simulations.
Experiment 1

- The simulation was run ten times for each of the four methods and each of the three values of N.
- The number of individuals in the group, K, is 10, and the number of alternatives, N, is 10, 20, or 30.
- To examine this method of consensus formation, the results of the simulations were compared.

Methods of consensus formation
- Method 0: Input values for \( c_{ij} \) at random.
- Consensus formation by search(\( S = 100, 1000, 10000 \)).
Result 1 (AFI)

Table I

<table>
<thead>
<tr>
<th></th>
<th>N = 10</th>
<th>N = 20</th>
<th>N = 30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ave. ± S. D.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Method 0</td>
<td>0.50±0.0208</td>
<td>0.50±0.0092</td>
<td>0.50±0.0096</td>
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<tr>
<td>S = 100</td>
<td>0.62±0.0113</td>
<td>0.62±0.0089</td>
<td>0.63±0.0086</td>
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<tr>
<td>S = 1000</td>
<td>0.64±0.0163</td>
<td>0.64±0.0065</td>
<td>0.64±0.0062</td>
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<tr>
<td>S = 10000</td>
<td>0.65±0.0139</td>
<td>0.64±0.0062</td>
<td>0.65±0.0061</td>
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</tbody>
</table>

Fig. 2

The random results and the high-AFI results from the consensus formation by search were chosen at random to fill unknown (i, j) elements of matrix C.

In consensus formation by search, we can predict that there is a cost associated with increasing the number of candidate solutions to a number sufficient for attaining a high degree of agreement.

However, for an effective search, the time required to generate candidate solutions becomes a problem.
Experiment 2

- The simulation was run ten times for each of the four methods and each of the three values of N.
- The number of individuals in the group, K, is 10, and the number of alternatives, N, is 10, 20, or 30.
- To examine this method of consensus formation, the results of the simulations were compared.

Methods of consensus formation

- **Method 1:** The opinion of one person, with many others in agreement, is assumed to become the dominant value of $c_{ij}$ in a group.
- **Method 2:** A probability is calculated by dividing the number of people in agreement with an opinion by the total number of people. The value of $c_{ij}$ is decided based on the prevalence of the opinion.
- **Method 3:** A value representing a minority opinion is assumed to be the value of $c_{ij}$, in contrast with method 1.
- **Consensus formation by search** ($S = 10000$).
Consensus formation by search yielded a better result than method 2 at $N = 10$, but method 2 performs better than the search at $N = 20$ and $N = 30$. This is because of, for the search, the search space increases as the number of choices increases, but the range of possible solutions does not change.

Method 1 achieved a higher score than any of the other methods or the search ($S = 10000$).

Consensus formation by method 1 got the best result out of all the methods that showed good results so that the opinion of one person becomes the consensus.
Result 2 (Consensus arrival rate)

Table III

<table>
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<tr>
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<th>N = 10</th>
<th>N = 20</th>
<th>N = 30</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ave. ± S. D.</td>
<td>Ave. ± S. D.</td>
<td>Ave. ± S. D.</td>
</tr>
<tr>
<td>Method 1</td>
<td>0.68±0.0175</td>
<td>0.70±0.0093</td>
<td>0.72±0.0071</td>
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<tr>
<td>Method 2</td>
<td>0.62±0.0149</td>
<td>0.65±0.0138</td>
<td>0.66±0.0113</td>
</tr>
<tr>
<td>Method 3</td>
<td>0.41±0.0490</td>
<td>0.48±0.0516</td>
<td>0.53±0.0331</td>
</tr>
</tbody>
</table>

Fig. 4

The consensus arrival rate results were the opposite of agreement degree AFI results.
Method 3 provided a better result than any of the other methods (Table III, Fig. 4).

This may indicate that the values that an consensus is fixed at by progress, one consensus increase without an opinion of the majority being reflected.

When the number of choices increases, the consensus arrival rate decreases. This may be due to an increase in strong connections between choices when the total number of choices increases; as a result, a higher number of values can be determined by implication at a single time.
Discussion

- It is the value that broke the agreement number of times that the consensus arrival rate was provided with the number that the number of opposite angle ingredients is subtracted from square, and was found of choices.

- An agreement result will be arise early; therefore, the value of the consensus arrival rate, which reflects how long it takes to reach consensus, is small.
Conclusion

- We suggest a group decision-making support model that uses the structural modeling method, FISM.
- We performed simulations, and examined the relationship between individual preferences and the group consensus.
- Consensus formation by fuzzy models of choice good relations is suggested for future study.